39. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

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PART I: STANDARD MODEL PROCESSES

Setting aside leptoproduction (for which, see Sec. 16 of this *Review*), the cross sections of primary interest are those with light incident particles, e^+e^-, $\gamma\gamma$, $q\overline{q}$, gq , gg, *etc.*, where g and q represent gluons and light quarks. The produced particles include both light particles and heavy ones - t, W, Z, and the Higgs boson H. We provide the production cross sections calculated within the Standard Model for several such processes.

39.1. Resonance Formation

Resonant cross sections are generally described by the Breit-Wigner formula (Sec. 16 of this *Review*).

$$\sigma(E) = \frac{2J+1}{(2S_1+1)(2S_2+1)}\frac{4\pi}{k^2}\left[\frac{\Gamma^2/4}{(E-E_0)^2+\Gamma^2/4}\right]B_{in}B_{out}, \tag{39.1}$$

where E is the c.m. energy, J is the spin of the resonance, and the number of polarization states of the two incident particles are $2S_1+1$ and $2S_2+2$. The c.m. momentum in the initial state is k, E_0 is the c.m. energy at the resonance, and Γ is the full width at half maximum height of the resonance. The branching fraction for the resonance into the initial-state channel is B_{in} and into the final-state channel is B_{out}. For a narrow resonance, the factor in square brackets may be replaced by $\pi\Gamma\delta(E-E_0)/2$.

39.2. Production of light particles

The production of point-like, spin-1/2 fermions in e^+e^- annihilation through a virtual photon, $e^+e^- \to \gamma^* \to f\overline{f}$, at c.m. energy squared s is given by

$$\frac{d\sigma}{d\Omega} = N_c\frac{\alpha^2}{4s}\beta\big[1+\cos^2\theta+(1-\beta^2)\sin^2\theta\big]Q_f^2\ , \tag{39.2}$$

where β is v/c for the produced fermions in the c.m., θ is the c.m. scattering angle, and Q_f is the charge of the fermion. The factor N_c is 1 for charged leptons and 3 for quarks. In the ultrarelativistic limit, $\beta \to 1$,

$$\sigma = N_c Q_f^2\frac{4\pi\alpha^2}{3s} = N_c Q_f^2\frac{86.8\ nb}{s(\mathrm{GeV}^2)^2}\ . \tag{39.3}$$

The cross section for the annihilation of a $q\overline{q}$ pair into a distinct pair $q'\overline{q}'$ through a gluon is completely analogous up to color factors, with the replacement $\alpha \to \alpha_s$. Treating all quarks as massless, averaging over the colors of the initial quarks and defining $t = -s\sin^2(\theta/2)$, $u = -s\cos^2(\theta/2)$, one finds [1]

$$\frac{d\sigma}{d\Omega}(q\overline{q} \to q'\overline{q}') = \frac{\alpha^2}{9s}\frac{t^2+u^2}{s^2}\ . \tag{39.4}$$

Crossing symmetry gives

$$\frac{d\sigma}{d\Omega}(qq' \to qq') = \frac{\alpha^2}{9s}\frac{s^2+u^2}{t^2} \; . \tag{39.5}$$

If the quarks q and q' are identical, we have

$$\frac{d\sigma}{d\Omega}(q\overline{q} \to q\overline{q}) = \frac{\alpha_s^2}{9s}\left[\frac{t^2+u^2}{s^2} + \frac{s^2+u^2}{t^2} - \frac{2u^2}{3st}\right] \; , \tag{39.6}$$

and by crossing

$$\frac{d\sigma}{d\Omega}(qq \to qq) = \frac{\alpha_s^2}{9s}\left[\frac{t^2+s^2}{u^2} + \frac{s^2+u^2}{t^2} - \frac{2s^2}{3ut}\right] . \tag{39.7}$$

Annihilation of e^+e^- into $\gamma\gamma$ has the cross section

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\frac{u^2+t^2}{tu} \; . \tag{39.8}$$

The related QCD process also has a triple-gluon coupling. The cross section is

$$\frac{d\sigma}{d\Omega}(q\overline{q} \to gg) = \frac{8\alpha_s^2}{27s}(t^2+u^2)\left(\frac{1}{tu} - \frac{9}{4s^2}\right) \; . \tag{39.9}$$

The crossed reactions are

$$\frac{d\sigma}{d\Omega}(qg \to qg) = \frac{\alpha_s^2}{9s}(s^2+u^2)(-\frac{1}{su} + \frac{9}{4t^2}) \tag{39.10}$$

and

$$\frac{d\sigma}{d\Omega}(gg \to q\overline{q}) = \frac{\alpha_s^2}{24s}(t^2+u^2)(\frac{1}{tu} - \frac{9}{4s^2}) \; . \tag{39.11}$$

Finally,

$$\frac{d\sigma}{d\Omega}(gg \to gg) = \frac{9\alpha_s^2}{8s}(3 - \frac{ut}{s^2} - \frac{su}{t^2} - \frac{st}{u^2}) \; . \tag{39.12}$$

39.3. Hadroproduction of heavy quarks

For hadroproduction of heavy quarks $Q = c,\ b,\ t$, it is important to include mass effects in the formulae. For $q\bar{q} \to Q\bar{Q}$, one has

$$\frac{d\sigma}{d\Omega}(q\bar{q} \to Q\bar{Q}) = \frac{\alpha_s^2}{9s^3}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right], \tag{39.13}$$

while for $gg \to Q\bar{Q}$ one has

$$\begin{aligned}\frac{d\sigma}{d\Omega}(gg \to Q\bar{Q}) = \frac{\alpha_s^2}{32s}\Bigg[&\frac{6}{s^2}(m_Q^2 - t)(m_Q^2 - u) - \frac{m_Q^2(s - 4m_Q^2)}{3(m_Q^2 - t)(m_Q^2 - u)} + \\ &\frac{4}{3}\frac{(m_Q^2 - t)(m_Q^2 - u) - 2m_Q^2(m_Q^2 + t)}{(m_Q^2 - t)^2} + \frac{4}{3}\frac{(m_Q^2 - t)(m_Q^2 - u) - 2m_Q^2(m_Q^2 + u)}{(m_Q^2 - u)^2} \\ &- 3\frac{(m_Q^2 - t)(m_Q^2 - u) + m_Q^2(u - t)}{s(m_Q^2 - t)} - 3\frac{(m_Q^2 - t)(m_Q^2 - u) + m_Q^2(t - u)}{s(m_Q^2 - u)}\Bigg].\end{aligned} \tag{39.14}$$

39.4. Production of Weak Gauge Bosons

39.4.1. *W and Z resonant production* :

Resonant production of a single W or Z is governed by the partial widths

$$\Gamma(W \to \ell_i \overline{\nu}_i) = \frac{\sqrt{2}G_F m_W^3}{12\pi} \tag{39.15}$$

$$\Gamma(W \to q_i \overline{q}_j) = 3\frac{\sqrt{2}G_F |V_{ij}|^2 m_W^3}{12\pi} \tag{39.16}$$

$$\begin{aligned}\Gamma(Z \to f\overline{f}) = {} & N_c\frac{\sqrt{2}G_F m_Z^3}{6\pi} \\ & \times \left[(T_3 - Q_f \sin^2\theta_W)^2 + (Q_f \sin\theta_W)^2\right].\end{aligned} \tag{39.17}$$

The weak mixing angle is θ_W. The CKM matrix elements are indicated by V_{ij} and N_c is 3 for $q\overline{q}$ final states and 1 for leptonic final states.

The full differential cross section for $f_i\overline{f}_j \to (W, Z) \to f_{i'}\overline{f}_{j'}$ is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega} = {} & \frac{N_c^f}{N_c^i} \cdot \frac{1}{256\pi^2 s} \cdot \frac{s^2}{(s - M^2)^2 + s\Gamma^2} \\ & \times \Big[(L^2 + R^2)(L'^2 + R'^2)(1 + \cos^2\theta) \\ & + (L^2 - R^2)(L'^2 - R'^2)2\ cos\theta\Big]\end{aligned} \tag{39.18}$$

where M is the mass of the W or Z. The couplings for the W are $L = (8G_F m_W^2/\sqrt{2})^{1/2} V_{ij}/\sqrt{2}; R = 0$ where V_{ij} is the corresponding CKM matrix element, with an analogous expression for L' and R'. For Z, the couplings are $L = (8G_F m_Z^2/\sqrt{2})^{1/2}(T_3 - \sin^2\theta_W Q); R = -(8G_F m_Z^2/\sqrt{2})^{1/2}\sin^2\theta_W Q$, where T_3 is the weak isospin of the initial left-handed fermion and Q is the initial fermion's electric charge. The expressions for L' and R' are analogous. The color factors $N_c^{i,f}$ are 3 for initial or final quarks and 1 for initial or final leptons.

39.4.2. *Production of pairs of weak gauge bosons* :

The cross section for $f\overline{f} \to W^+W^-$ is given in term of the couplings of the left-handed and right-handed fermion f, $\ell = 2(T_3 - Qx_W)$, $r = -2Qx_W$, where T_3 is the third component of weak isospin for the left-handed f, Q is its electric charge (in units of the proton charge), and $x_W = \sin^2\theta_W$:

$$\begin{aligned}\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{N_c s^2}\Bigg\{&\left[\left(Q + \frac{\ell + r}{4x_W}\frac{s}{s - m_Z^2}\right)^2 + \left(\frac{\ell + r}{4x_W}\frac{s}{s - m_Z^2}\right)^2\right]A(s,t,u) \\ &+ \frac{1}{2x_W}\left(Q + \frac{\ell}{2x_W}\frac{s}{s - m_Z^2}\right)(\Theta(-Q)I(s,t,u) - \Theta(Q)I(s,u,t)) \\ &+ \frac{1}{8x_W^2}(\Theta(-Q)E(s,t,u) + \Theta(Q)E(s,u,t))\Bigg\}\,, \end{aligned} \tag{39.19}$$

where $\Theta(x)$ is 1 for $x > 0$ and 0 for $x < 0$, and where

$$\begin{aligned} A(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} - \frac{m_W^2}{s} + 3\frac{m_W^4}{s^2}\right) + \frac{s}{m_W^2} - 4, \\ I(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} - \frac{m_W^2}{2s} - \frac{m_W^4}{st}\right) + \frac{s}{m_W^2} - 2 + 2\frac{m_W^2}{t}, \\ E(s,t,u) &= \left(\frac{tu}{m_W^4} - 1\right)\left(\frac{1}{4} + \frac{m_W^2}{t}\right) + \frac{s}{m_W^2}, \end{aligned} \tag{39.20}$$

and s, t, u are the usual Mandelstam variables with $s = (p_f + p_{\overline{f}})^2, t = (p_f - p_{W^-})^2, u = (p_f - p_{W^+})^2$. The factor N_c is 3 for quarks and 1 for leptons.

The analogous cross-section for $q_i\overline{q}_j \to W^\pm Z^0$ is

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2|V_{ij}|^2}{6s^2x_W^2}\Bigg\{\left(\frac{1}{s - m_W^2}\right)^2\Bigg[\left(\frac{9 - 8x_W}{4}\right)\left(ut - m_W^2 m_Z^2\right)$$

$$+ (8x_W - 6)\, s \left(m_W^2 + m_Z^2\right)\Bigg]$$
$$+ \left[\frac{ut - m_W^2 m_Z^2 - s(m_W^2 + m_Z^2)}{s - m_W^2}\right]\left[\frac{\ell_j}{t} - \frac{\ell_i}{u}\right]$$
$$+ \frac{ut - m_W^2 m_Z^2}{4(1 - x_W)}\left[\frac{\ell_j^2}{t^2} + \frac{\ell_i^2}{u^2}\right] + \frac{s(m_W^2 + m_Z^2)}{2(1 - x_W)}\frac{\ell_i \ell_j}{tu}\Bigg\}, \tag{39.21}$$

where ℓ_i and ℓ_j are the couplings of the left-handed q_i and q_j as defined above. The CKM matrix element between q_i and q_j is V_{ij}.

The cross section for $q_i \overline{q}_i \to Z^0 Z^0$ is

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{96}\frac{\ell_i^4 + r_i^4}{x_W^2(1 - x_W^2)s^2}\left[\frac{t}{u} + \frac{u}{t} + \frac{4m_Z^2 s}{tu} - m_Z^4\left(\frac{1}{t^2} + \frac{1}{u^2}\right)\right]. \tag{39.22}$$

39.5. Production of Higgs Bosons

39.5.1. *Resonant Production* :

The Higgs boson of the Standard Model can be produced resonantly in the collisions of quarks, leptons, W or Z bosons, gluons, or photons. The production cross section is thus controlled by the partial width of the Higgs boson into the entrance channel and its total width. The branching fractions for the Standard Model Higgs boson are shown in Fig. 1 of the "Searches for Higgs bosons" review in the Particle Listings section, as a function of the Higgs boson mass. The partial widths are given by the relations

$$\Gamma(H \to f\overline{f}) = \frac{G_F m_f^2 m_H N_c}{4\pi\sqrt{2}}\left(1 - 4m_f^2/m_H^2\right)^{3/2}, \tag{39.23}$$
$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3 \beta_W}{32\pi\sqrt{2}}\left(4 - 4a_W + 3a_W^2\right), \tag{39.24}$$
$$\Gamma(H \to ZZ) = \frac{G_F m_H^3 \beta_Z}{64\pi\sqrt{2}}\left(4 - 4a_Z + 3a_Z^2\right), \tag{39.25}$$

where N_c is 3 for quarks and 1 for leptons and where $a_W = 1 - \beta_W^2 = 4m_W^2/m_H^2$ and $a_Z = 1 - \beta_Z^2 = 4m_Z^2/m_H^2$. The decay to two gluons proceeds through quark loops, with the t quark dominating [2]. Explicitly,

$$\Gamma(H \to gg) = \frac{\alpha_s^2 G_F m_H^3}{36\pi^3\sqrt{2}}\left|\sum_q I(m_q^2/m_H^2)\right|^2, \tag{39.26}$$

where $I(z)$ is complex for $z < 1/4$. For $z < 2 \times 10^{-3}$, $|I(z)|$ is small so the light quarks contribute negligibly. For $m_H < 2m_t$, $z > 1/4$ and

$$I(z) = 3\left[2z + 2z(1-4z)\left(\sin^{-1}\frac{1}{2\sqrt{z}}\right)^2\right], \tag{39.27}$$

which has the limit $I(z) \to 1$ as $z \to \infty$.

39.5.2. *Higgs Boson Production in W^* and Z^* decay* :

The Standard Model Higgs boson can be produced in the decay of a virtual W or Z ("Higgstrahlung") [3,4]: In particular, if k is the c.m. momentum of the Higgs boson,

$$\sigma(q_i\overline{q}_j \to WH) = \frac{\pi\alpha^2|V_{ij}|^2}{36\sin^4\theta_W}\frac{2k}{\sqrt{s}}\frac{k^2+3m_W^2}{(s-m_W^2)^2} \tag{39.28}$$

$$\sigma(f\overline{f} \to ZH) = \frac{2\pi\alpha^2(\ell_f^2+r_f^2)}{48N_c\sin^4\theta_W\cos^4\theta_W}\frac{2k}{\sqrt{s}}\frac{k^2+3m_Z^2}{(s-m_Z^2)^2}, \tag{39.29}$$

where ℓ and r are defined as above.

39.5.3. *W and Z Fusion* :

Just as high-energy electrons can be regarded as sources of virtual photon beams, at very high energies they are sources of virtual W and Z beams. For Higgs boson production, it is the longitudinal components of the Ws and Zs that are important [5]. The distribution of longitudinal Ws carrying a fraction y of the electron's energy is [6]

$$f(y) = \frac{g^2}{16\pi^2}\frac{1-y}{y}, \tag{39.30}$$

where $g = e/\sin\theta_W$. In the limit $s \gg m_H \gg m_W$, the partial decay rate is $\Gamma(H \to W_L W_L) = (g^2/64\pi)(m_H^3/m_W^2)$ and in the equivalent W approximation [7]

$$\begin{aligned}\sigma(e^+e^- \to \overline{\nu}_e\nu_e H) = {} & \frac{1}{16m_W^2}\left(\frac{\alpha}{\sin^2\theta_W}\right)^3 \\ & \times\left[\left(1+\frac{m_H^2}{s}\right)\log\frac{s}{m_H^2} - 2 + 2\frac{m_H^2}{s}\right].\end{aligned} \tag{39.31}$$

There are significant corrections to this relation when m_H is not large compared to m_W [8]. For $m_H = 150$ GeV, the estimate is too high by 51% for $\sqrt{s} = 1000$ GeV, 32% too high at $\sqrt{s} = 2000$ GeV, and 22% too high at $\sqrt{s} = 4000$ GeV. Fusion of ZZ to make a Higgs boson can be treated similarly. Identical formulae apply for Higgs production in the collisions of quarks whose charges permit the emission of a W^+ and a W^-, except that QCD corrections and CKM matrix elements are required. Even in the absence of QCD corrections, the fine-structure constant ought to be evaluated at the scale of the collision, say m_W. All quarks contribute to the ZZ fusion process.

39.6. Inclusive hadronic reactions

One-particle inclusive cross sections $Ed^3\sigma/d^3p$ for the production of a particle of momentum p are conveniently expressed in terms of rapidity y (see above) and the momentum p_T transverse to the beam direction (in the c.m.):

$$E\frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi\, dy\, p_T\, dp_T^2} \ . \tag{39.32}$$

In appropriate circumstances, the cross section may be decomposed as a partonic cross section multiplied by the probabilities of finding partons of the prescribed momenta:

$$\sigma_{\text{hadronic}} = \sum_{ij} \int dx_1\ dx_2\ f_i(x_1)\ f_j(x_2)\ d\widehat{\sigma}_{\text{partonic}} \ , \tag{39.33}$$

The probability that a parton of type i carries a fraction of the incident particle's that lies between x_1 and $x_1 + dx_1$ is $f_i(x_1)dx_1$ and similarly for partons in the other incident particle. The partonic collision is specified by its c.m. energy squared $\hat{s} = x_1 x_2 s$ and the momentum transfer squared $\hat{t}$. The final hadronic state is more conveniently specified by the rapidities y_1, y_2 of the two jets resulting from the collision and the transverse momentum p_T. The connection between the differentials is

$$dx_1 dx_2 d\hat{t} = dy_1 dy_2 \frac{\hat{s}}{s} dp_T^2, \tag{39.34}$$

so that

$$\frac{d^3\sigma}{dy_1 dy_2 dp_T^2} = \frac{\hat{s}}{s}\left[f_i(x_1) f_j(x_2) \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}) + f_i(x_2) f_j(x_1) \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, \hat{u}, \hat{t}) \right] , \tag{39.35}$$

where we have taken into account the possibility that the incident parton types might arise from either incident particle. The second term should be dropped if the types are identical: $i = j$.

39.7. Two-photon processes

In the Weizsäcker-Williams picture, a high-energy electron beam is accompanied by a spectrum of virtual photons of energies ω and invariant-mass squared $q^2 = -Q^2$, for which the photon number density is

$$dn = \frac{\alpha}{\pi}\left[1 - \frac{\omega}{E} + \frac{\omega^2}{E^2} - \frac{m_e^2\,\omega^2}{Q^2E^2}\right]\frac{d\omega}{\omega}\frac{dQ^2}{Q^2}, \tag{39.36}$$

where E is the energy of the electron beam. The cross section for $e^+e^- \to e^+e^-X$ is then [9]

$$d\sigma_{e^+e^-\to e^+e^-X}(s) = dn_1 dn_2 d\sigma_{\gamma\gamma\to X}(W^2), \tag{39.37}$$

where $W^2 = m_X^2$. Integrating from the lower limit $Q^2 = m_e^2\dfrac{\omega_i^2}{E_i(E_i-\omega_i)}$ to a maximum Q^2 gives

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-X}(s) = {} & \frac{\alpha^2}{\pi^2}\int_{z_{th}}^{1}\frac{dz}{z} \\ & \times\left[\left(\ln\frac{Q^2_{max}}{zm_e^2} - 1\right)^2 f(z) + \frac{1}{3}(\ln z)^3\right]\sigma_{\gamma\gamma\to X}(zs),\end{aligned} \tag{39.38}$$

where

$$f(z) = \left(1 + \tfrac{1}{2}z\right)^2\ln(1/z) - \tfrac{1}{2}(1-z)(3+z). \tag{39.39}$$

The appropriate value of Q^2_{max} depends on the properties of the produced system X. For production of hadronic systems, $Q^2_{max} \approx m_\rho^2$, while for lepton-pair production, $Q^2 \approx W^2$. For production of a resonance with spin $J \neq 1$, we have

$$\begin{aligned}\sigma_{e^+e^-\to e^+e^-R}(s) = {} & (2J+1)\frac{8\alpha^2\Gamma_{R\to\gamma\gamma}}{m_R^3} \\ & \times\left[f(m_R^2/s)\left(\ln\frac{m_V^2 s}{m_e^2 m_R^2} - 1\right)^2 - \frac{1}{3}\left(\ln\frac{s}{M_R^2}\right)^3\right]\end{aligned} \tag{39.40}$$

where m_V is the mass that enters into the form factor for the $\gamma\gamma \to R$ transition, typically m_ρ.

PART II: PROCESSES BEYOND THE STANDARD MODEL

39.8. Production of supersymmetric particles

In supersymmetric (SUSY) theories (see Supersymmetric Particle Searches in this *Review*), every boson has a fermionic superpartner, and every fermion has a bosonic superpartner. The minimal supersymmetric Standard Model (MSSM) is a direct supersymmetrization of the Standard Model (SM), although a second Higgs doublet is needed to avoid triangle anomalies [10]. Under *soft* SUSY breaking, superpartner masses are lifted above the SM particle masses. In weak scale SUSY, the superpartners are invoked to stabilize the weak scale under radiative corrections, so the superpartners are expected to have masses of order the TeV scale.

39.8.1. *Gluino and squark production* :

The superpartners of gluons are the color octet, spin$-\frac{1}{2}$ gluinos ($\tilde{g}$), while each helicity component of quark flavor has a spin-0 *squark* partner, *e.g.* $\tilde{q}_L$ and $\tilde{q}_R$. Third generation left- and right- squarks are expected to have large mixing, resulting in mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$, with $m_{\tilde{q}_1} < m_{\tilde{q}_2}$ (here, q denotes any of the SM flavors of quarks and $\tilde{q}_i$ the corresponding flavor and type ($i = L, R$ or $1, 2$) of squark). Gluino pair production ($\tilde{g}\tilde{g}$) takes place via either glue-glue or quark-antiquark annihilation [11].

The subprocess cross sections are usually presented as differential distributions in the Mandelstam variables s, t and u. Note that for a $2 \to 2$ scattering subprocess $ab \to cd$, the Mandelstam variable $s = (p_a + p_b)^2 = (p_c + p_d)^2$, where p_a is the 4-momentum of particle a, and so forth. The variable $t = (p_c - p_a)^2$, where c and a are taken conventionally to be the most similar particles in the subprocess. The variable u would then be equal to $(p_d - p_a)^2$. Note that since s, t and u are squares of 4-vectors, they are invariants in any inertial reference frame.

Gluino pair production at hadron colliders is described by:

$$\begin{aligned}\frac{d\sigma}{dt}(gg \to \tilde{g}\tilde{g}) = \frac{9\pi\alpha_s^2}{4s^2}\Bigg\{&\frac{2(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u)}{s^2} + \frac{(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u) - 2m_{\tilde{g}}^2(m_{\tilde{g}}^2 + t)}{(m_{\tilde{g}}^2 - t)^2}\\ &+ \frac{(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u) - 2m_{\tilde{g}}^2(m_{\tilde{g}}^2 + u)}{(m_{\tilde{g}}^2 - u)^2} + \frac{m_{\tilde{g}}^2(s - 4m_{\tilde{g}}^2)}{(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u)}\\ &- \frac{(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u) + m_{\tilde{g}}^2(u - t)}{s(m_{\tilde{g}}^2 - t)} - \frac{(m_{\tilde{g}}^2 - t)(m_{\tilde{g}}^2 - u) + m_{\tilde{g}}^2(t - u)}{s(m_{\tilde{g}}^2 - u)}\Bigg\},\end{aligned} \tag{39.41}$$

where α_s is the strong fine structure constant. Also,

$$\begin{aligned}\frac{d\sigma}{dt}(q\bar{q} \to \tilde{g}\tilde{g}) = \frac{8\pi\alpha_s^2}{9s^2}\Bigg\{&\frac{4}{3}\left(\frac{m_{\tilde{g}}^2 - t}{m_{\tilde{q}}^2 - t}\right)^2 + \frac{4}{3}\left(\frac{m_{\tilde{g}}^2 - u}{m_{\tilde{q}}^2 - u}\right)^2\\ &+ \frac{3}{s^2}\left[(m_{\tilde{g}}^2 - t)^2 + (m_{\tilde{g}}^2 - u)^2 + 2m_{\tilde{g}}^2 s\right] - 3\frac{\left[(m_{\tilde{g}}^2 - t)^2 + m_{\tilde{g}}^2 s\right]}{s(m_{\tilde{q}}^2 - t)}\end{aligned}$$

$$- 3 \frac{\left[(m_{\tilde{g}}^2 - u)^2 + m_{\tilde{g}}^2 s\right]}{s(m_{\tilde{q}}^2 - u)} + \frac{1}{3} \frac{m_{\tilde{g}}^2 s}{(m_{\tilde{q}}^2 - t)(m_{\tilde{q}}^2 - u)} \Bigg\} . \tag{39.42}$$

Gluinos can also be produced in association with squarks: $\tilde{g}\tilde{q}_i$ production, where $\tilde{q}_i$ represents any of the various types (left-, right- or mixed) and flavors of squarks. The subprocess cross section is independent of whether the squark is the right-, left- or mixed type:

$$\frac{d\sigma}{dt}(gq \to \tilde{g}\tilde{q}_i) = \frac{\pi\alpha_s^2}{24s^2} \frac{\left[\frac{16}{3}(s^2 + (m_{\tilde{q}_i}^2 - u)^2) + \frac{4}{3}s(m_{\tilde{q}_i}^2 - u)\right]}{s(m_{\tilde{g}}^2 - t)(m_{\tilde{q}_i}^2 - u)^2}$$

$$\times \left((m_{\tilde{g}}^2 - u)^2 + (m_{\tilde{q}_i}^2 - m_{\tilde{g}}^2)^2 + \frac{2sm_{\tilde{g}}^2(m_{\tilde{q}_i}^2 - m_{\tilde{g}}^2)}{(m_{\tilde{g}}^2 - t)} \right) . \tag{39.43}$$

There are many different subprocesses for production of squark pairs. Since left- and right- squarks generally have different masses and different decay patterns, we present the differential cross section for each subprocess of $\tilde{q}_i$ ($i = L,\ R$ or $1,\ 2$) separately. (In early literature, the following formulae were often combined into a single equation which didn't differentiate the various squark types.) The result for $gg \to \tilde{q}_i\bar{\tilde{q}}_i$ is:

$$\frac{d\sigma}{dt}(gg \to \tilde{q}_i\bar{\tilde{q}}_i) = \frac{\pi\alpha_s^2}{4s^2} \Bigg\{ \frac{1}{3}\left(\frac{m_{\tilde{q}}^2 + t}{m_{\tilde{q}}^2 - t}\right)^2 + \frac{1}{3}\left(\frac{m_{\tilde{q}}^2 + u}{m_{\tilde{q}}^2 - u}\right)^2$$

$$+ \frac{3}{32s^2}\left(8s(4m_{\tilde{q}}^2 - s) + 4(u - t)^2\right) + \frac{7}{12}$$

$$- \frac{1}{48} \frac{(4m_{\tilde{q}}^2 - s)^2}{(m_{\tilde{q}}^2 - t)(m_{\tilde{q}}^2 - u)}$$

$$+ \frac{3}{32} \frac{\left[(t - u)(4m_{\tilde{q}}^2 + 4t - s) - 2(m_{\tilde{q}}^2 - u)(6m_{\tilde{q}}^2 + 2t - s)\right]}{s(m_{\tilde{q}}^2 - t)}$$

$$+ \frac{3}{32} \frac{\left[(u - t)(4m_{\tilde{q}}^2 + 4u - s) - 2(m_{\tilde{q}}^2 - t)(6m_{\tilde{q}}^2 + 2u - s)\right]}{s(m_{\tilde{q}}^2 - u)}$$

$$+ \frac{7}{96} \frac{\left[4m_{\tilde{q}}^2 + 4t - s\right]}{m_{\tilde{q}}^2 - t} + \frac{7}{96} \frac{\left[4m_{\tilde{q}}^2 + 4u - s\right]}{m_{\tilde{q}}^2 - u} \Bigg\} , \tag{39.44}$$

which has an obvious $u \leftrightarrow t$ symmetry.

For $q\bar{q} \to \tilde{q}_i\bar{\tilde{q}}_i$ with the same initial and final state flavors, we have

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{q}_i\bar{\tilde{q}}_i) = \frac{2\pi\alpha_s^2}{9s^2} \left\{ \frac{1}{(t - m_{\tilde{g}}^2)^2} + \frac{2}{s^2} - \frac{2/3}{s(t - m_{\tilde{g}}^2)} \right\}$$

$$\times \left[-st - (t - m_{\tilde{q}_i}^2)^2\right] , \tag{39.45}$$

while if initial and final state flavors are different ($q\bar{q} \to \tilde{q}'_i \bar{\tilde{q}}'_i$) we instead have

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{q}'_i \bar{\tilde{q}}'_i) = \frac{4\pi\alpha_s^2}{9s^4}\left[-st - (t - m_{\tilde{q}'_i}^2)^2\right]. \tag{39.46}$$

If the two initial state quarks are of different flavors, then we have

$$\frac{d\sigma}{dt}(q\bar{q}' \to \tilde{q}_i \bar{\tilde{q}}'_i) = \frac{2\pi\alpha_s^2}{9s^2}\frac{-st - (t - m_{\tilde{q}_i}^2)^2}{(t - m_{\tilde{g}}^2)^2}. \tag{39.47}$$

If the initial quarks are of different flavor and final state squarks are of different type ($i \neq j$) then

$$\frac{d\sigma}{dt}(q\bar{q}' \to \tilde{q}_i \bar{\tilde{q}}'_j) = \frac{2\pi\alpha_s^2}{9s^2}\frac{m_{\tilde{g}}^2 s}{(t - m_{\tilde{g}}^2)^2}. \tag{39.48}$$

For same-flavor initial state quarks, but final state unlike-type squarks, we also have

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{q}_i \bar{\tilde{q}}_j) = \frac{2\pi\alpha_s^2}{9s^2}\frac{m_{\tilde{g}}^2 s}{(t - m_{\tilde{g}}^2)^2}. \tag{39.49}$$

There also exist cross sections for quark-quark annihilation to squark pairs. For same flavor quark-quark annihilation to same flavor/same type final state squarks,

$$\frac{d\sigma}{dt}(qq \to \tilde{q}_i \tilde{q}_i) = \\ = \frac{\pi\alpha_s^2}{9s^2} m_{\tilde{g}}^2 s \left\{\frac{1}{(t - m_{\tilde{g}}^2)^2} + \frac{1}{(u - m_{\tilde{g}}^2)^2} - \frac{2/3}{(t - m_{\tilde{g}}^2)(u - m_{\tilde{g}}^2)}\right\}, \tag{39.50}$$

while if the final type squarks are different ($i \neq j$), we have

$$\frac{d\sigma}{dt}(qq \to \tilde{q}_i \tilde{q}_j) = \\ \frac{2\pi\alpha_s^2}{9s^2}\left\{\frac{[-st - (t - m_{\tilde{q}_i}^2)(t - m_{\tilde{q}_j}^2)]}{(t - m_{\tilde{g}}^2)} + \frac{[-su - (u - m_{\tilde{q}_i}^2)(u - m_{\tilde{q}_j}^2)]}{(u - m_{\tilde{g}}^2)}\right\}. \tag{39.51}$$

If initial/final state flavors are different, but final state squark types are the same, then

$$\frac{d\sigma}{dt}(qq' \to \tilde{q}_i \tilde{q}'_i) = \frac{2\pi\alpha_s^2}{9s^2}\frac{m_{\tilde{g}}^2 s}{(t - m_{\tilde{g}}^2)^2}. \tag{39.52}$$

If initial quark flavors are different and final squark types are different, then

$$\frac{d\sigma}{dt}(qq' \to \tilde{q}_i \tilde{q}'_j) = \frac{2\pi\alpha_s^2}{9s^2}\frac{-st - (t - m_{\tilde{q}_i}^2)(t - m_{\tilde{q}_j}^2)}{(t - m_{\tilde{g}}^2)^2}. \tag{39.53}$$

39.8.2. *Gluino and squark associated production* :

In the MSSM, the charged spin-$\frac{1}{2}$ winos and higgsinos mix to make chargino states $\chi_1^{\pm}$ and $\chi_2^{\pm}$, with $m_{\chi_1^{\pm}} < m_{\chi_2^{\pm}}$. The spin$-\frac{1}{2}$ neutral bino, wino and higgsino fields mix to give four neutralino mass eigenstates $\chi^0_{1,2,3,4}$ ordered according to mass. We sometimes denote the charginos and neutralinos collectively as -inos for notational simplicity

For gluino and squark production in association with charginos and neutralinos [12], the quark-squark-neutralino couplings* are defined by the interaction Lagrangian terms $\mathcal{L}_{\tilde{f}f\tilde{\chi}_i^0} = \left[iA^f_{\tilde{\chi}_i^0}\tilde{f}_L^\dagger\bar{\tilde{\chi}}_i^0 P_L f + iB^f_{\tilde{\chi}_i^0}\tilde{f}_R^\dagger\bar{\tilde{\chi}}_i^0 P_R f + \text{h.c.}\right]$, where $A^f_{\tilde{\chi}_i^0}$ and $B^f_{\tilde{\chi}_i^0}$ are coupling constants involving gauge couplings, neutralino mixing elements and in the case of third generation fermions, Yukawa couplings. Their form depends on the conventions used for setting up the MSSM Lagrangian, and can be found in various reviews [13] and textbooks [14,15]. P_L and P_R are the usual left- and right- spinor projection operators and f denotes any of the SM fermions u, d, e, $\nu_e, \cdots$. The fermion-sfermion- chargino couplings have the form $\mathcal{L} = \left[iA^d_{\tilde{\chi}_i^-}\tilde{u}_L^\dagger\overline{\tilde{\chi}_i^-}P_L d + iA^u_{\tilde{\chi}_i^-}\tilde{d}_L^\dagger\overline{\tilde{\chi}_i^c}P_L u + \text{h.c.}\right]$ for u and d quarks, where the $A^d_{\tilde{\chi}_i^-}$ and $A^u_{\tilde{\chi}_i^-}$ couplings are again convention-dependent, and can be found in textbooks. The superscript c denotes "charge conjugate spinor", defined by $\psi^c \equiv C\bar{\psi}^T$.

The subprocess cross sections for chargino-squark associated production occur via squark exchange and are given by

$$\frac{d\sigma}{dt}(\bar{u}g \to \tilde{\chi}_i^-\bar{\tilde{d}}_L) = \frac{\alpha_s}{24s^2}|A^u_{\tilde{\chi}_i^-}|^2\psi(m_{\tilde{d}_L}, m_{\tilde{\chi}_i^-}, t), \tag{39.54}$$

$$\frac{d\sigma}{dt}(dg \to \tilde{\chi}_i^-\tilde{u}_L) = \frac{\alpha_s}{24s^2}|A^d_{\tilde{\chi}_i^-}|^2\psi(m_{\tilde{u}_L}, m_{\tilde{\chi}_i^-}, t), \tag{39.55}$$

while neutralino-squark production is given by

$$\frac{d\sigma}{dt}(qg \to \tilde{\chi}_i^0\tilde{q}) = \frac{\alpha_s}{24s^2}\left(|A^q_{\tilde{\chi}_i^0}|^2 + |B^q_{\tilde{\chi}_i^0}|^2\right)\psi(m_{\tilde{q}}, m_{\tilde{\chi}_i^0}, t), \tag{39.56}$$

where

$$\psi(m_1, m_2, t) = \frac{s + t - m_1^2}{2s} - \frac{m_1^2(m_2^2 - t)}{(m_1^2 - t)^2} + \frac{t(m_2^2 - m_1^2) + m_2^2(s - m_2^2 + m_1^2)}{s(m_1^2 - t)}. \tag{39.57}$$

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* The couplings $A^f_{\tilde{\chi}_i^0}$ and $B^f_{\tilde{\chi}_i^0}$ are given explicitly in Ref. 15 in Eq. (8.87). Also, the couplings $A^d_{\tilde{\chi}_i^-}$ and $A^u_{\tilde{\chi}_i^-}$ are given in Eq. (8.93). The couplings X_i^j and Y_i^j are given by Eq. (8.103), while the x_i and y_i couplings are given in Eq. (8.100). Finally, the couplings W_{ij} are given in Eq. (8.101).

Here, the variable t is given by the square of "squark-minus-quark" four-momentum. The neutralino-gluino associated production cross section also occurs via squark exchange and is given by

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{\chi}_i^0 \tilde{g}) = \frac{\alpha_s}{18s^2}\left(|A^q_{\tilde{\chi}_i^0}|^2 + |B^q_{\tilde{\chi}_i^0}|^2\right)\left[\frac{(m^2_{\tilde{\chi}_i^0} - t)(m^2_{\tilde{g}} - t)}{(m^2_{\tilde{q}} - t)^2}\right.$$
$$\left. + \frac{(m^2_{\tilde{\chi}_i^0} - u)(m^2_{\tilde{g}} - u)}{(m^2_{\tilde{q}} - u)^2} - \frac{2\eta_i\eta_{\tilde{g}} m_{\tilde{g}} m_{\tilde{\chi}_i^0} s}{(m^2_{\tilde{q}} - t)(m^2_{\tilde{q}} - u)}\right], \tag{39.58}$$

where η_i is the sign of the neutralino mass eigenvalue and $\eta_{\tilde{g}}$ is the sign of the gluino mass eigenvalue. We also have chargino-gluino associated production:

$$\frac{d\sigma}{dt}(\bar{u}d \to \tilde{\chi}_i^- \tilde{g}) = \frac{\alpha_s}{18s^2}\left[|A^u_{\tilde{\chi}_i^-}|^2 \frac{(m^2_{\tilde{\chi}_i^-} - t)(m^2_{\tilde{g}} - t)}{(m^2_{\tilde{d}_L} - t)^2}\right.$$
$$\left. + |A^d_{\tilde{\chi}_i^-}|^2 \frac{(m^2_{\tilde{\chi}_i^-} - u)(m^2_{\tilde{g}} - u)}{(m^2_{\tilde{u}_L} - u)^2} + \frac{2\eta_{\tilde{g}} Re(A^u_{\tilde{\chi}_i^-} A^d_{\tilde{\chi}_i^-}) m_{\tilde{g}} m_{\tilde{\chi}_i} s}{(m^2_{\tilde{d}_L} - t)(m^2_{\tilde{u}_L} - u)}\right], \tag{39.59}$$

where $\hat{t} = (\tilde{g} - d)^2$ and in the third term one must take the real part of the in general complex coupling constant product.

39.8.3. *Slepton and sneutrino production* :

The subprocess cross section for $\tilde{\ell}_L \bar{\tilde{\nu}}_{\ell_L}$ production ($\ell = e$ or μ) occurs via s-channel W exchange and is given by

$$\frac{d\sigma}{dt}(d\bar{u} \to \tilde{\ell}_L \bar{\tilde{\nu}}_{\ell_L}) = \frac{g^4 |D_W(s)|^2}{192\pi s^2}\left(tu - m^2_{\tilde{\ell}_L} m^2_{\tilde{\nu}_{\ell_L}}\right), \tag{39.60}$$

where $D_W(s) = 1/(s - M_W^2 + iM_W\Gamma_W)$ is the W-boson propagator denominator. The production of $\tilde{\tau}_1 \bar{\tilde{\nu}}_\tau$ is given as above, but replacing $m_{\tilde{\ell}_L} \to m_{\tilde{\tau}_1}$, $m_{\tilde{\nu}_{\ell_L}} \to m_{\tilde{\nu}_\tau}$ and multiplying by an overall factor of $\cos^2\theta_\tau$ (where θ_τ is the tau-slepton mixing angle). Similar substitutions hold for $\tilde{\tau}_2 \bar{\tilde{\nu}}_\tau$ production, except the overall factor is $\sin^2\theta_\tau$.

The subprocess cross section for $\tilde{\ell}_L \bar{\tilde{\ell}}_L$ production occurs via s-channel γ and Z exchange, and depends on the neutral current interaction, with fermion couplings to γ and Z^0 given by $\mathcal{L}_{\text{neutral}} = -eq_f \bar{f}\gamma^\mu f A_\mu + e\bar{f}\gamma^\mu(\alpha_f + \beta_f\gamma_5) f Z_\mu$ (with values of q_f, α_f, and β_f given in Table 39.1.

The subprocess cross section is given by

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{\ell}_L \bar{\tilde{\ell}}_L) = \frac{e^4}{24\pi s^2}\left(tu - m^4_{\tilde{\ell}_L}\right) \times$$

Table 39.1: The constants α_f and β_f that appear in in the SM neutral current Lagrangian. Here $t \equiv \tan\theta_W$ and $c \equiv \cot\theta_W$.

f	q_f	α_f	β_f
ℓ	-1	$\frac{1}{4}(3t-c)$	$\frac{1}{4}(t+c)$
ν_ℓ	0	$\frac{1}{4}(t+c)$	$-\frac{1}{4}(t+c)$
u	$\frac{2}{3}$	$-\frac{5}{12}t+\frac{1}{4}c$	$-\frac{1}{4}(t+c)$
d	$-\frac{1}{3}$	$\frac{1}{12}t-\frac{1}{4}c$	$\frac{1}{4}(t+c)$

$$\left\{\frac{q_\ell^2 q_q^2}{s^2} + (\alpha_\ell-\beta_\ell)^2(\alpha_q^2+\beta_q^2)|D_Z(s)|^2 + \frac{2q_\ell q_q \alpha_q(\alpha_\ell-\beta_\ell)(s-M_Z^2)}{s}|D_Z(s)|^2\right\}, \quad (39.61)$$

where $D_Z(s) = 1/(s-M_Z^2+iM_Z\Gamma_Z)$. The cross section for sneutrino production is given by the same formula, but with α_ℓ, β_ℓ, q_ℓ and $m_{\tilde{\ell}_L}$ replaced by α_ν, β_ν, 0 and $m_{\tilde{\nu}_L}$, respectively. The cross section for $\tilde{\tau}_1\bar{\tilde{\tau}}_1$ production is obtained by replacing $m_{\tilde{\ell}_L} \to m_{\tilde{\tau}_1}$ and $\beta_\ell \to \beta_\ell\cos 2\theta_\tau$. The cross section for $\tilde{\ell}_R\bar{\tilde{\ell}}_R$ production is given by substituting $\alpha_\ell-\beta_\ell \to \alpha_\ell+\beta_\ell$ and $m_{\tilde{\ell}_L} \to m_{\tilde{\ell}_R}$ in the equation above. The cross section for $\tilde{\tau}_2\bar{\tilde{\tau}}_2$ production is obtained from the formula for $\tilde{\ell}_R\bar{\tilde{\ell}}_R$ production by replacing $m_{\tilde{\ell}_R} \to m_{\tilde{\tau}_2}$ and $\beta_\ell \to \beta_\ell\cos 2\theta_\tau$.

Finally, the cross section for $\tilde{\tau}_1\bar{\tilde{\tau}}_2$ production occurs only via Z exchange, and is given by

$$\frac{d\sigma}{dt}(q\bar{q} \to \tilde{\tau}_1\bar{\tilde{\tau}}_2) = \frac{d\sigma}{dt}(q\bar{q} \to \bar{\tilde{\tau}}_1\tilde{\tau}_2) =$$

$$\frac{e^4}{24\pi s^2}(\alpha_q^2+\beta_q^2)\beta_\ell^2\sin^2 2\theta_\tau|D_Z(s)|^2(ut-m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2). \quad (39.62)$$

39.8.4. *Chargino and neutralino pair production* :

39.8.4.1. $\tilde{\chi}_i^- \tilde{\chi}_j^0$ *production:*

The subprocess cross section for $d\bar{u} \to \tilde{\chi}_i^- \tilde{\chi}_j^0$ depends on Lagrangian couplings $\mathcal{L}_{W\bar{u}d} = -\frac{g}{\sqrt{2}}\bar{u}\gamma_\mu P_L d W^{+\mu}\ +\text{h.c.}$, $\mathcal{L}_{W\tilde{\chi}_i^-\tilde{\chi}_j^0} = -g(-i)^{\theta_j}\overline{\tilde{\chi}^-}_i[X_i^j + Y_i^j\gamma_5]\gamma_\mu\tilde{\chi}_j^0 W^{-\mu}\ +\text{h.c.}$, $\mathcal{L}_{q\tilde{q}\tilde{\chi}_i^-} = iA^d_{\tilde{\chi}_i^-}\tilde{u}_L^\dagger\overline{\tilde{\chi}_i^-}P_L d + iA^u_{\tilde{\chi}_i^-}\tilde{d}_L^\dagger\overline{\tilde{\chi}_i^c}P_L u\ +\text{h.c.}$ and $\mathcal{L}_{q\tilde{q}\tilde{\chi}_j^0} = iA^q_{\tilde{\chi}_j^0}\tilde{q}_L^\dagger\overline{\tilde{\chi}^0}_j P_L q\ +\text{h.c.}$. Contributing diagrams include W exchange and also $\tilde{d}_L$ and $\tilde{u}_L$ squark exchange. The X_i^j and Y_i^j couplings are new, and again convention-dependent: the cross section formulae works if the interaction Lagrangian is written in the above form, so that the couplings can be suitably extracted. The term $\theta_j = 0$ (1) if $m_{\tilde{\chi}_j^0} > 0$ (< 0); it comes about because the neutralino field must be re-defined by a $-i\gamma_5$ transformation if its mass eigenvalue is negative [15]. The subprocess cross section is given in terms of dot products of four momenta, where particle labels are used to denote their four-momenta; note that all mass terms in the cross section formulae are positive definite, so that the signs of mass eigenstates have been absorbed into the Lagrangian couplings, as for instance in Ref. [15]. We then have

$$\frac{d\sigma}{dt}(d\bar{u} \to \tilde{\chi}_i^- \tilde{\chi}_j^0) = \frac{1}{192\pi s^2}\left[T_W + T_{\tilde{d}_L} + T_{\tilde{u}_L} + T_{W\tilde{d}_L} + T_{W\tilde{u}_L} + T_{\tilde{d}_L\tilde{u}_L}\right] \tag{39.63}$$

where

$$\begin{aligned} T_W = 8g^4|D_W(s)|^2 \Big\{ &[X_i^{j2} + Y_i^{j2}](\tilde{\chi}_j^0 \cdot d\tilde{\chi}_i^- \cdot \overline{u} + \tilde{\chi}_j^0 \cdot \overline{u}\tilde{\chi}_i^- \cdot d) \\ &+ 2(X_i^j Y_i^j)(\tilde{\chi}_j^0 \cdot d\tilde{\chi}_i^- \cdot \overline{u} - \tilde{\chi}_j^0 \cdot \overline{u}\tilde{\chi}_i^- \cdot d) + [X_i^{j2} - Y_i^{j2}] m_{\tilde{\chi}_i^-} m_{\tilde{\chi}_j^0} d \cdot \overline{u} \Big\}, \end{aligned} \tag{39.64}$$

$$T_{\tilde{d}_L} = \frac{4|A^u_{\tilde{\chi}_i^-}|^2 |A^d_{\tilde{\chi}_j^0}|^2}{[(\tilde{\chi}_i^- - \overline{u})^2 - m^2_{\tilde{d}_L}]^2}\ d \cdot \tilde{\chi}_j^0 \tilde{\chi}_i^- \cdot \overline{u}, \tag{39.65}$$

$$T_{\tilde{u}_L} = \frac{4|A^d_{\tilde{\chi}_i^-}|^2 |A^u_{\tilde{\chi}_j^0}|^2}{[(\tilde{\chi}_j^0 - \overline{u})^2 - m^2_{\tilde{u}_L}]^2}\ \overline{u} \cdot \tilde{\chi}_j^0 \tilde{\chi}_i^- \cdot d \tag{39.66}$$

$$\begin{aligned} T_{W\tilde{d}_L} = &\frac{-\sqrt{2}g^2 Re[A^{d*}_{\tilde{\chi}_j^0} A^u_{\tilde{\chi}_i^-}(-i)^{\theta_j}](s - M_W^2)|D_W(s)|^2}{(\tilde{\chi}_i^- - \overline{u})^2 - m^2_{\tilde{d}_L}} \\ &\times \left\{ 8(X_i^j + Y_i^j)\tilde{\chi}_j^0 \cdot d\overline{u} \cdot \tilde{\chi}_i^- + 4(X_i^j - Y_i^j) m_{\tilde{\chi}_i^-} m_{\tilde{\chi}_j^0} d \cdot \overline{u} \right\} \end{aligned} \tag{39.67}$$

$$T_{W\tilde{u}_L} = \frac{\sqrt{2}g^2 Re[A^{d*}_{\tilde{\chi}_i^-} A^u_{\tilde{\chi}_j^0}(-i)^{\theta_j}](s - M_W^2)|D_W(s)|^2}{(\tilde{\chi}_j^0 - \overline{u})^2 - m^2_{\tilde{u}_L}}$$
$$\times \left\{ 8(X_i^j - Y_i^j)\tilde{\chi}_j^0 \cdot \overline{u} d \cdot \tilde{\chi}_i^- + 4(X_i^j + Y_i^j) m_{\tilde{\chi}_i^-} m_{\tilde{\chi}_j^0} d \cdot \overline{u} \right\} \tag{39.68}$$

and

$$T_{\tilde{d}_L\tilde{u}_L} = -\frac{4Re[A^d_{\tilde{\chi}_j^0} A^{u*}_{\tilde{\chi}_i^-} A^{d*}_{\tilde{\chi}_i^-} A^u_{\tilde{\chi}_j^0}] m_{\tilde{\chi}_i^-} m_{\tilde{\chi}_j^0} d \cdot \overline{u}}{[(\tilde{\chi}_i^- - \overline{u})^2 - m^2_{\tilde{d}_L}][(\tilde{\chi}_j^0 - \overline{u})^2 - m^2_{\tilde{u}_L}]}. \tag{39.69}$$

39.8.4.2. *Chargino pair production:*

The subprocess cross section for $d\bar{d} \to \tilde{\chi}_i^- \tilde{\chi}_i^+$ $(i = 1, 2)$ depends on Lagrangian couplings $\mathcal{L} = e\overline{\tilde{\chi}_i^-}\gamma_\mu \tilde{\chi}_i^- A^\mu - e\cot\theta_W \overline{\tilde{\chi}_i^-}\gamma_\mu(x_i - y_i\gamma_5)\tilde{\chi}_i^- Z^\mu$ and also $\mathcal{L} \ni iA^d_{\tilde{\chi}_i^-}\tilde{u}_L^\dagger \overline{\tilde{\chi}_i^-} P_L d + iA^u_{\tilde{\chi}_i^-}\tilde{d}_L^\dagger \overline{\tilde{\chi}_i^{-c}} P_L u + \text{h.c.}$. Contributing diagrams include s-channel γ, Z^0 exchange and t-channel $\tilde{u}_L$ exchange [16,17]. The couplings x_i and y_i are again new and as usual convention-dependent.

The subprocess cross section is given by

$$\frac{d\sigma}{dt}(d\bar{d} \to \tilde{\chi}_i^- \tilde{\chi}_i^+) = \frac{1}{192\pi s^2}\left[T_\gamma + T_Z + T_{\tilde{u}_L} + T_{\gamma Z} + T_{\gamma\tilde{u}_L} + T_{Z\tilde{u}_L}\right] \tag{39.70}$$

where

$$T_\gamma = \frac{32e^4 q_d^2}{s^2}\left[d \cdot \tilde{\chi}_i^+ \overline{d} \cdot \tilde{\chi}_i^- + d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+ + m^2_{\tilde{\chi}_i^-} d \cdot \overline{d}\right] \tag{39.71}$$

$$T_Z = 32e^4\cot^2\theta_W|D_Z(s)|^2\left\{(\alpha_d^2 + \beta_d^2)(x_i^2 + y_i^2)\left[d \cdot \tilde{\chi}_i^+ \overline{d} \cdot \tilde{\chi}_i^- + d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+ + m^2_{\tilde{\chi}_i^-} d \cdot \overline{d}\right]\right.$$
$$\left. \mp 4\alpha_d\beta_d x_i y_i\left[d \cdot \tilde{\chi}_i^+ \overline{d} \cdot \tilde{\chi}_i^- - d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+\right] - 2y_i^2(\alpha_d^2 + \beta_d^2) m^2_{\tilde{\chi}_i^-} d \cdot \overline{d}\right\}, \tag{39.72}$$

$$T_{\tilde{u}_L} = \frac{4|A^d_{\tilde{\chi}_i^-}|^4}{[(d - \tilde{\chi}_i^-)^2 - m^2_{\tilde{u}_L}]^2}\, d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+ \tag{39.73}$$

$$T_{\gamma Z} = \frac{64e^4\cot\theta_W q_d(s - M_Z^2)|D_Z(s)|^2}{s} \times$$
$$\left\{\alpha_d x_i \left(d \cdot \tilde{\chi}_i^+ \overline{d} \cdot \tilde{\chi}_i^- + d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+ + m^2_{\tilde{\chi}_i^-} d \cdot \overline{d}\right) \pm \beta_d y_i \left(d \cdot \tilde{\chi}_i^- \overline{d} \cdot \tilde{\chi}_i^+ - d \cdot \tilde{\chi}_i^+ \overline{d} \cdot \tilde{\chi}_i^-\right)\right\} \tag{39.74}$$

$$T_{\gamma\tilde{u}_L} = \mp\frac{8e^2 q_d}{s}\frac{|A^d_{\tilde{\chi}_i^-}|^2}{[(d-\tilde{\chi}_i^-)^2 - m^2_{\tilde{u}_L}]}\left\{2\overline{d}\cdot\tilde{\chi}_i^+ d\cdot\tilde{\chi}_i^- + m^2_{\tilde{\chi}_i^-} d\cdot\overline{d}\right\} \tag{39.75}$$

and

$$T_{Z\tilde{u}_L} = \mp 8e^2\cot\theta_W |D_Z(s)|^2 \frac{|A^d_{\tilde{\chi}_i^-}|^2 (s-M_Z^2)}{[(d-\tilde{\chi}_i^-)^2 - m^2_{\tilde{u}_L}]}(\alpha_d - \beta_d)$$
$$\times\left\{2(x_i \mp y_i) d\cdot\tilde{\chi}_i^-\overline{d}\cdot\tilde{\chi}_i^+ + m^2_{\tilde{\chi}_i^-}(x_i \pm y_i) d\cdot\overline{d}\right\} \tag{39.76}$$

using the upper of the sign choices.

The cross section for $u\overline{u}\to\tilde{\chi}_i^+\tilde{\chi}_i^-$ can be obtained from the above by replacing $\alpha_d\to\alpha_u$, $\beta_d\to\beta_u$, $q_d\to q_u$, $\tilde{u}_L\to\tilde{d}_L$, $A^d_{\tilde{\chi}_i^-}\to A^u_{\tilde{\chi}_i^-}$, $d\to\overline{u}$, $\overline{d}\to u$ and adopting the lower of the sign choices everywhere.

The cross section for $q\bar{q}\to\tilde{\chi}_1^-\tilde{\chi}_2^+$, $\tilde{\chi}_1^+\tilde{\chi}_2^-$ can occur via Z and $\tilde{q}_L$ exchange. It is usually much smaller than $\tilde{\chi}_{1,2}^-\tilde{\chi}_{1,2}^+$ production, so the cross section will not be presented here. It can be found in Appendix A of Ref. 15.

39.8.4.3. *Neutralino pair production:*

Neutralino pair production via $q\bar{q}$ fusion takes place via s-channel Z exchange plus t- and u-channel left- and right- squark exchange (5 diagrams) [17,18]. The Lagrangian couplings (see previous footnote*) needed include terms given above plus terms of the form $\mathcal{L} = W_{ij}\overline{\tilde{\chi}^0}_i\gamma_\mu(\gamma_5)^{\theta_i+\theta_j+1}\tilde{\chi}_j^0 Z^\mu$. The couplings W_{ij} depend only on the *higgsino* components of the neutralinos i and j. The subprocess cross section is given by:

$$\frac{d\sigma}{dt}(q\bar{q}\to\tilde{\chi}_i^0\tilde{\chi}_j^0) = \frac{1}{192\pi s^2}\left[T_Z + T_{\tilde{q}_L} + T_{\tilde{q}_R} + T_{Z\tilde{q}_L} + T_{Z\tilde{q}_R}\right] \tag{39.77}$$

where

$$T_Z = 128e^2|W_{ij}|^2(\alpha_q^2+\beta_q^2)|D_Z(s)|^2\left[q\cdot\tilde{\chi}_i^0\bar{q}\cdot\tilde{\chi}_j^0 + q\cdot\tilde{\chi}_j^0\bar{q}\cdot\tilde{\chi}_i^0 - \eta_i\eta_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} q\cdot\bar{q}\right], \tag{39.78}$$

$$T_{\tilde{q}_L} = 4|A^q_{\tilde{\chi}_i^0}|^2|A^q_{\tilde{\chi}_j^0}|^2\left\{\frac{q\cdot\tilde{\chi}_i^0\bar{q}\cdot\tilde{\chi}_j^0}{[(\tilde{\chi}_i^0-q)^2-m^2_{\tilde{q}_L}]^2} + \frac{q\cdot\tilde{\chi}_j^0\bar{q}\cdot\tilde{\chi}_i^0}{[(\tilde{\chi}_j^0-q)^2-m^2_{\tilde{q}_L}]^2}\right.$$
$$\left. - \eta_i\eta_j\frac{m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}q\cdot\bar{q}}{[(\tilde{\chi}_i^0-q)^2-m^2_{\tilde{q}_L}][(\tilde{\chi}_j^0-q)^2-m^2_{\tilde{q}_L}]}\right\} \tag{39.79}$$

$$T_{\tilde{q}_R} = 4|B^q_{\tilde{\chi}_i^0}|^2|B^q_{\tilde{\chi}_j^0}|^2\left\{\frac{q\cdot\tilde{\chi}_i^0\bar{q}\cdot\tilde{\chi}_j^0}{[(\tilde{\chi}_i^0-q)^2-m^2_{\tilde{q}_R}]^2} + \frac{q\cdot\tilde{\chi}_j^0\bar{q}\cdot\tilde{\chi}_i^0}{[(\tilde{\chi}_j^0-q)^2-m^2_{\tilde{q}_R}]^2}\right.$$
$$\left. - \eta_i\eta_j\frac{m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}q\cdot\bar{q}}{[(\tilde{\chi}_i^0-q)^2-m^2_{\tilde{q}_R}][(\tilde{\chi}_j^0-q)^2-m^2_{\tilde{q}_R}]}\right\} \tag{39.80}$$

$$T_{Z\tilde{q}_L} = 16e(\alpha_q - \beta_q)(s - M_Z^2)|D_Z(s)|^2 \left\{ \frac{Re(W_{ij} A^{q*}_{\tilde{\chi}_i^0} A^q_{\tilde{\chi}_j^0})}{[(\tilde{\chi}_i^0 - q)^2 - m^2_{\tilde{q}_L}]} \left[2q \cdot \tilde{\chi}_i^0 \bar{q} \cdot \tilde{\chi}_j^0 - \eta_i \eta_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} q \cdot \bar{q} \right] \right.$$
$$\left. + \eta_i \eta_j \frac{Re(W_{ij} A^q_{\tilde{\chi}_i^0} A^{q*}_{\tilde{\chi}_j^0})}{[(\tilde{\chi}_j^0 - q)^2 - m^2_{\tilde{q}_L}]} \left[2q \cdot \tilde{\chi}_j^0 \bar{q} \cdot \tilde{\chi}_i^0 - \eta_i \eta_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} q \cdot \bar{q} \right] \right\} \tag{39.81}$$

$$T_{Z\tilde{q}_R} = 16e(\alpha_q + \beta_q)(s - M_Z^2)|D_Z(s)|^2 \left\{ \frac{Re(W_{ij} B^{q*}_{\tilde{\chi}_i^0} B^q_{\tilde{\chi}_j^0})}{[(\tilde{\chi}_i^0 - q)^2 - m^2_{\tilde{q}_R}]} \left[2q \cdot \tilde{\chi}_i^0 \bar{q} \cdot \tilde{\chi}_j^0 - \eta_i \eta_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} q \cdot \bar{q} \right] \right.$$
$$\left. - \frac{Re(W_{ij} B^q_{\tilde{\chi}_i^0} B^{q*}_{\tilde{\chi}_j^0})}{[(\tilde{\chi}_j^0 - q)^2 - m^2_{\tilde{q}_R}]} \left[2q \cdot \tilde{\chi}_j^0 \bar{q} \cdot \tilde{\chi}_i^0 - \eta_i \eta_j m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} q \cdot \bar{q} \right] \right\} . \tag{39.82}$$

As before, $\eta_i = \pm 1$ corresponding to whether the neutralino mass eigenvalue is positive or negative. When $i = j$ in the above formula, one must remember to integrate over just 2π steradians of solid angle to avoid double counting in the total cross section.

39.9. Universal extra dimensions

In the Universal Extra Dimension (UED) model of Ref. [19](see Ref. [20] for a review of models with extra spacetime dimensions), the Standard Model is embedded in a five dimensional theory, where the fifth dimension is compactified on an S_1/Z_2 orbifold. Each SM chirality state is then the zero mode of an infinite tower of Kaluza-Klein excitations labelled by $n = 0 - \infty$. A KK parity is usually assumed to hold, where each state is assigned KK-parity $P = (-1)^n$. If the compactification scale is around a TeV, then the $n = 1$ (or even higher) KK modes may be accessible to collider searches.

Of interest for hadron colliders are the production of massive $n \geq 1$ quark or gluon pairs. These production cross sections have been calculated in Ref. [21,22]. We list here results for the $n = 1$ case only with $M_1 = 1/R$ (R is the compactification radius) and s, t and u are the usual Mandelstam variables; more general formulae can be found in Ref. [22]. The superscript $*$ stands for any KK excited state, while $\bullet$ stands for left chirality states and $\circ$ stands for right chirality states.

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} T \tag{39.83}$$

where

$$T(q\bar{q} \to g^* g^*) = \frac{2g_s^4}{27} \left[M_1^2 \left(-\frac{4s^3}{t'^2 u'^2} + \frac{57s}{t'u'} - \frac{108}{s} \right) + \frac{20s^2}{t'u'} - 93 + \frac{108t'u'}{s^2} \right] \tag{39.84}$$

and

$$T(gg \to g^* g^*) =$$

$$\frac{9g_s^4}{27}\left[3M_1^4\frac{s^2+t'^2+u'^2}{t'^2u'^2}-3M_1^2\frac{s^2+t'^2+u'^2}{st'u'}+1+\frac{(s^2+t'^2+u'^2)^3}{4s^2t'^2u'^2}-\frac{t'u'}{s^2}\right] \tag{39.85}$$

where $t' = t - M_1^2$ and $u' = u - M_1^2$.

Also,

$$T(q\bar{q}\to q_1^{*'}\bar{q}_1^{*'}) = \frac{4g_s^4}{9}\left[\frac{2M_1^2}{s}+\frac{t'^2+u'^2}{s^2}\right],$$

$$T(q\bar{q}\to q_1^{*}\bar{q}_1^{*}) = \frac{g_2^4}{9}\left[2M_1^2\left(\frac{4}{s}+\frac{s}{t'^2}-\frac{1}{t'}\right)+\frac{23}{6}+\frac{2s^2}{t'^2}+\frac{8s}{3t'}+\frac{6t'}{s}+\frac{8t'^2}{s^2}\right],$$

$$T(qq\to q_1^{*}q_1^{*}) = \frac{g_s^4}{27}\left[M_1^2\left(6\frac{t'}{u'^2}+6\frac{u'}{t'^2}-\frac{s}{t'u'}\right)+2\left(3\frac{t'^2}{u'^2}+3\frac{u'^2}{t'^2}+4\frac{s^2}{t'u'}-5\right)\right],$$

$$T(gg\to q_1^{*}\bar{q}_1^{*}) = g_s^4\left[M_1^4\frac{-4}{t'u'}\left(\frac{s^2}{6t'u'}-\frac{3}{8}\right)+M_1^2\frac{4}{s}\left(\frac{s^2}{6t'u'}-\frac{3}{8}\right)+\frac{s^2}{6t'u'}-\frac{17}{24}+\frac{3t'u'}{4s^2}\right],$$

$$T(gq\to g^{*}q_1^{*}) = \frac{-g_s^4}{3}\left[\frac{5s^2}{12t'^2}+\frac{s^3}{t'^2u'}+\frac{11su'}{6t'^2}+\frac{5u'^2}{12t'^2}+\frac{u'^3}{st'^2}\right],$$

$$T(q\bar{q}'\to q_1^{*}\bar{q}_1^{*'}) = \frac{g_s^4}{18}\left[4M_1^4\frac{s}{t'^2}+5+4\frac{s^2}{t'^2}+8\frac{s}{t'}\right],$$

$$T(qq'\to q_1^{*}q_1^{*'}) = \frac{2g_s^4}{9}\left[-M_1^2\frac{s}{t'^2}+\frac{1}{4}+\frac{s^2}{t'^2}\right],$$

$$T(qq\to q_1^{\bullet}q_1^{\circ}) = \frac{g_s^4}{9}\left[M_1^2\left(\frac{2s^3}{t'^2u'^2}-\frac{4s}{t'u'}\right)+2\frac{s^4}{t'^2u'^2}-8\frac{s^2}{t'u'}+5\right],$$

$$T(q\bar{q}'\to q_1^{\bullet}\bar{q}_1^{'\circ}) = \frac{g_s^4}{9}\left[2M_1^2\left(\frac{1}{t'}+\frac{u'}{t'^2}\right)+\frac{5}{2}+\frac{4u'}{t'}+\frac{2u'^2}{t'^2}\right],$$

and

$$T(qq'\to q_1^{\bullet}q_1^{'\circ}) = \frac{g_s^4}{9}\left[-2M_1^2\left(\frac{1}{t'}+\frac{u'}{t'^2}\right)+\frac{1}{2}+\frac{2u'^2}{t'^2}\right].$$

39.10. Large extra dimensions

In the ADD theory [23] with large extra dimensions (LED), the SM particles are confined to a 3-brane, while gravity propagates in the bulk. It is assumed that the n extra dimensions are compactified on an n-dimensional torus of volume $(2\pi r)^n$, so that the fundamental $4+n$ dimensional Planck scale M_* is related to the usual 4-dimensional Planck scale M_{Pl} by $M_{Pl}^2 = M_*^{n+2}(2\pi r)^n$. If $M_* \sim 1$ TeV, then the $M_W - M_{Pl}$ hierarchy problem is just due to gravity propagating in the large extra dimensions.

In these theories, the KK-excited graviton states $G^n_{\mu\nu}$ for $n = 1 - \infty$ can be produced at collider experiments. The graviton couplings to matter are suppressed by $1/M_{Pl}$, so that graviton emission cross sections $d\sigma/dt \sim 1/M_{Pl}^2$. However, the mass splittings between the excited graviton states can be tiny, so the graviton eigenstates are usually approximated by a continuum distribution. A summation (integration) over all allowed graviton emissions ends up cancelling the $1/M_{Pl}^2$ factor, so that observable cross section rates can be attained. Some of the fundamental production formulae for a KK graviton (denoted G) of mass m at hadron colliders include the subprocesses

$$\frac{d\sigma_m}{dt}(f\bar{f} \to \gamma G) = \frac{\alpha Q_f^2}{16N_f}\frac{1}{sM_{Pl}^2}F_1(\frac{t}{s},\frac{m^2}{s}), \tag{39.86}$$

where Q_f is the charge of fermion f and N_f is the number of QCD colors of f. Also,

$$\frac{d\sigma_m}{dt}(q\bar{q} \to gG) = \frac{\alpha_s}{36}\frac{1}{sM_{Pl}^2}F_1(\frac{t}{s},\frac{m^2}{s}), \tag{39.87}$$

$$\frac{d\sigma_m}{dt}(qg \to qG) = \frac{\alpha_s}{96}\frac{1}{sM_{Pl}^2}F_2(\frac{t}{s},\frac{m^2}{s}), \tag{39.88}$$

$$\frac{d\sigma_m}{dt}(gg \to gG) = \frac{3\alpha_s}{16}\frac{1}{sM_{Pl}^2}F_3(\frac{t}{s},\frac{m^2}{s}), \tag{39.89}$$

where

$$\begin{aligned} F_1(x,y) =& \frac{1}{x(y-1-x)}\Big[-4x(1+x)(1+2x+2x^2)+ \\ & y(1+6x+18x^2+16x^3) - 6y^2x(1+2x) + y^3(1+4x)\Big] \end{aligned} \tag{39.90}$$

$$F_2(x,y) = -(y-1-x)F_1\left(\frac{x}{y-1-x},\frac{y}{y-1-x}\right) \tag{39.91}$$

and

$$\begin{aligned} F_3(x,y) =& \frac{1}{x(y-1-x)}\Big[1+2x+3x^2+2x^3+x^4 \\ & -2y(1+x^3)+3y^2(1+x^2)-2y^3(1+x)+y^4\Big]. \end{aligned} \tag{39.92}$$

These formulae must then be multiplied by the graviton density of states formula $dN = S_{n-1} \frac{M_{Pl}^2}{M_*^{n+2}} m^{n-1} dm$ to gain the cross section

$$\frac{d^2\sigma}{dtdm} = S_{n-1} \frac{M_{Pl}^2}{M_*^{n+2}} m^{n-1} \frac{d\sigma_m}{dt} \tag{39.93}$$

where $S_n = \frac{(2\pi)^{n/2}}{\Gamma(n/2)}$ is the surface area of an n-dimensional sphere of unit radius.

Virtual graviton processes can also be searched for at colliders. For instance, in Ref. [24] the cross section for Drell-Yan production of lepton pairs via gluon fusion was calculated, where it is found that, in the center-of-mass system

$$\frac{d\sigma}{dz}(gg \to \ell^+\ell^-) = \frac{\lambda^2 s^3}{64\pi M_*^8}(1 - z^2)(1 + z^2)$$

where $z = \cos\theta$ and λ is a model-dependent coupling constant ~ 1. Formulae for Drell-Yan production via $q\bar{q}$ fusion can also be found in Ref. [24,25].

39.11. Warped extra dimensions

In the Randall-Sundrum model [26] of warped extra dimensions, the arena for physics is a 5-d anti-deSitter (AdS_5) spacetime, for which a non-factorizable metric exists with a metric warp factor $e^{-2\sigma(\phi)}$. It is assumed that two opposite tension 3-branes exist within AdS_5 at the two ends of an S_1/Z_2 orbifold parametrized by co-ordinate ϕ which runs from $0 - \pi$. The 4-D solution of the Einstein equations yields $\sigma(\phi) = kr_c|\phi|$, where r_c is the compactification radius of the extra dimension and $k \sim M_{Pl}$. The 4-D effective action allows one to identify $\overline{M}_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kr_c\pi})$, where M is the 5-D Planck scale. Physical particles on the TeV scale (SM) brane have mass $m = e^{-kr_c\pi}m_0$, where m_0 is a fundamental mass of order the Planck scale. Thus, the weak scale-Planck scale hierarchy occurs due to the existence of the exponential warp factor if $kr_c \sim 12$.

In the simplest versions of the RS model, the TeV-scale brane contains only SM particles plus a tower of KK gravitons. The RS gravitons have mass $m_n = kx_n e^{-kr_c\pi}$, where the x_i are roots of Bessel functions $J_1(x_n) = 0$, with $x_1 \simeq 3.83$, $x_2 \simeq 7.02$ etc. While the RS zero-mode graviton couplings suppressed by $1/\overline{M}_{Pl}$ and are thus inconsequential for collider searches, the $n = 1$ and higher modes have couplings suppressed instead by $\Lambda_\pi = e^{-kr_c\pi}\overline{M}_{Pl} \sim TeV$. The $n = 1$ RS graviton should have width $\Gamma_1 = \rho m_1 x_1^2 (k/\overline{M}_{Pl})^2$, where ρ is a constant depending on how many decay modes are open. The formulae for dilepton production via virtual RS graviton exchange can be gained from the above formulae for the ADD scenario via the replacement [27]

$$\frac{\lambda}{M_*^4} \to \frac{i^2}{8\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + im_n\Gamma_n}. \tag{39.94}$$

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